Modeling the Geometry of the Quark Gluon Plasma

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Abstract. We extend the Djordjevic-Gyulassy-Levai-Vitev (DGLV) model, which describes jet energy dissipation in the quark-gluon plasma (QGP), a phenomenon observed in high-energy particle collisions at the LHC and RHIC. Derived from perturbative quantum chromodynamics (pQCD), the model presents complex challenges in extracting observable predictions. Our goal is to simplify the QGP's geometry while maintaining accuracy, enabling quantitative predictions comparable to experimental data. The focus is on capturing both hydrodynamic and geometric features of the QGP, with a simplification that allows numerical models to converge efficiently.

1. Introduction

Recent experimental findings at RHIC and the LHC involving heavy ion collisions (denoted as AA collisions) have revealed significant amounts of suppression in particles with high momentum ($\sim$ 5GeV) in the direction transverse to the beam axis (p_T) [1, 2, 3]. The suppression in the spectra of the high p_T particles can be attributed to the energy loss of partons (quarks or gluons) moving through a state of matter known as the quark gluon plasma, which forms following an AA collision. The techniques of perturbative QCD can be used to study the phenomenology of the high p_T suppression observed at RHIC and the LHC.

In this article, we will present the DGLV energy loss model along with its convolved radiative and collisional energy loss extensions of Wicks, Horowitz, Djordjevic, and Gyulassy (WHDG) [4, 5]. This framework allows us theoretical access to an observable known as the nuclear modification (R_{AA}) which is related to the energy loss of partons. Our particular work has been to highlight some of the difficulties of working with the *effective length* prescription provided by the WHDG model. In addition to this, we showcase a modification to the model that moves away from the effective length prescription.

2. DGLV Energy Loss Model

2.1. R_{AA}

The actual energy loss of the particles within the jets (a stream of high p_T particles) is not experimentally accessible. However, there are other (more statistical) methods for understanding how the partons within the jets lose energy. One method is by measuring the R_{AA} of incoming partons as a function of p_T. We will use an expression for the R_{AA} derived by Horowitz [6], this expression is:

$$R_{AA} \approx \int d\epsilon P(\epsilon)(1-\epsilon)^{n(p_T)-1} \tag{1}$$

Here, $P(\epsilon)$ is the probability of fractional energy loss occurring and $n(p_T)$ is a slowly varying function of p_T which is experimentally determined.

2.2. Energy Loss Formalism

2.2.1. Radiative Energy Loss Let us define the manuscript we will be using in this project, we will keep the same conventions as those found in [7]. $C_A = 3$ and $C_R = 4/3$ are the Casimir in the gluon representation and the Casimir in a representation related to the leading parton respectively while α_s is the strong coupling constant. The two momenta $\mathbf{k}$ and $\mathbf{q}$ are the transverse momentum of the radiated gluon and the transverse momentum exchanged between the radiated gluon and the medium respectively. We also have that M is the mass of the incident parton, L is the length of the brick of QGP, μ is the Debye mass and $m_g = \mu/\sqrt{2}$ is the mass of the gluon which is obtained by taking into the QCD analogue of the Ter-Mikayelian plasmon effect [8]. The quatity described by $\tilde{\rho}(\Delta z)$ is the density of scattering sites in the medium. The scattering centre density can be thought of as the density of the medium through which the parent parton is moving, each scattering centre can be seen as a static parton which the parent parton could potentially interact with. The scattering centre is assumed to be described by a Gyulassy-Wang Debye screened potential. Lastly, ϵ is the polarization of a massless boson. We will make use of the following short-hand to simplify our expressions

$$\begin{aligned}
\omega &= xE \\
\omega_0 &= \mathbf{k}^2/2\omega \\
\omega_1 &= (\mathbf{k}-\mathbf{q})^2/2\omega \\
\tilde{\omega} &= (m_g^2 + M^2x^2)/2\omega
\end{aligned}$$

With these conventions in place, the radiative energy loss of a single gluon emission takes the form

$$\begin{aligned}
\frac{dE_{rad}}{dx} &= \frac{C_R\alpha_s LE}{\pi}\int\frac{d^2\mathbf{q}}{\pi}\frac{\mu^2}{\lambda\left(\mu^2+\mathbf{q}^2\right)^2}\int\frac{d^2\mathbf{k}}{\pi}\int d\Delta z\tilde{\rho}(\Delta z) \\
&\times\left[-\frac{2\left(1-\cos\left\{\left(\omega_1+\tilde{\omega}_m\right)\Delta z\right\}\right)}{(\mathbf{k}-\mathbf{q})^2+m_g^2+x^2M^2}\left(\frac{(\mathbf{k}-\mathbf{q})\cdot\mathbf{k}}{\mathbf{k}^2+m_g^2+x^2M^2}-\frac{(\mathbf{k}-\mathbf{q})^2}{(\mathbf{k}-\mathbf{q})^2+m_g^2+x^2M^2}\right)\right. \\
&+\frac{1}{2}e^{-\mu_1\Delta z}\left\{\left(\frac{\mathbf{k}}{\mathbf{k}^2+m_g^2+x^2M^2}\right)^2\left(1-\frac{2C_R}{C_A}\right)\left(1-\cos\left\{\left(\omega_0+\tilde{\omega}_m\right)\Delta z\right\}\right)\right. \\
&\left.\left.+\frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{q})\left(\cos\left\{\left(\omega_0+\tilde{\omega}_m\right)\Delta z\right\}-\cos\left\{\left(\omega_0-\omega_1\right)\Delta z\right\}\right)}{\left(\mathbf{k}^2+m_g^2+x^2M^2\right)\left((\mathbf{k}-\mathbf{q})^2+m_g^2+x^2M^2\right)}\right\}\right].
\end{aligned} \tag{2}$$

The second line along with the prefactor in the first line in eq. (2) is the radiative energy loss derived by DGLV [4] and we shall refer to its contribution as *DGLV* contribution. The third and fourth lines, again along with the prefactor, is the short path length correction to the energy loss derived by Kolbe and Horowitz [7] and we shall refer to its contribution as the corretion term.

2.2.2. Effective Lengths The DGLV formalism works under the assumption that a drop of QGP forms in the shape of a brick, this drastically simplifies the analytics of the formalism. To capture the effects that a more realistically shaped drop of QGP would have on the energy loss of partons moving through it— while still working under the DGLV formalism — one can introduce an effective length into the energy loss calculation. The effective length allows us to assign a single number as a representation of the overall geomertry of the QGP and the particular path taken by the parton. The general structure of an effective length can be calculated as [9]:

$$L_{eff}(\mathbf{x}, \phi) = \frac{1}{N} \int_0^\infty d\tau \quad n(\mathbf{x} + \hat{\phi}\tau) \tag{3}$$

We explored a variety of different effective length prescriptions, all of which are variants of eq. (3). The hopes were to find some uniformity in the predictions made by the different effective length prescriptions, and to find a variant which agreed well with experiment. Below we present all six effective length prescriptions we explored. The hydrodynamics in the models presented here were generated by [10, 11]

Path Independent

$$L(\mathbf{x}, \phi, \tau_0) = \frac{1}{T^3_{eff}} \int_{\tau_0}^\infty dz T^3(\mathbf{x} + z\hat{\phi}, \tau_0) \tag{4}$$

$$T^3_{eff} = \left(\int d^2x T^6(\mathbf{x}, \tau_0) \right) \left(\int d^2x T^3(\mathbf{x}, \tau_0) \right)^{-1} \tag{5}$$

The *Path Independent* variant fixes the temperature distribution T at $t = \tau_0$ (τ_0 is the formation time of the medium) and normalizes with T^3_{eff} by integrating over the entire static medium, i.e. the normalization is path independent.

Path Dependent

$$L(\mathbf{x}, \phi, \tau_0) = \frac{1}{T^3_{eff}(\mathbf{x})} \int_{\tau_0}^\infty dt T^3(\mathbf{x} + z\hat{\phi}, \tau_0) \tag{6}$$

$$T^3_{eff}(\mathbf{x}) = \left(\int_{\tau_0}^\infty dt T^6(\mathbf{x} + z\hat{\phi}, \tau_0) \right) \left(\int_{\tau_0}^\infty dt T^3(\mathbf{x} + z\hat{\phi}, \tau_0) \right)^{-1} \tag{7}$$

The *Path Dependent* variant again fixes the temperature distribution T at $t = \tau_0$ but now normalizes with T^3_{eff} by integrating through a particular path, i.e. the normalization is path dependent.

Path Dependent - Bjorken

$$L(\mathbf{x}, \phi, \tau_0) = \frac{1}{T^3_{eff}(\mathbf{x})} \int_{\tau_0}^\infty dt T^3(\mathbf{x} + z\hat{\phi}, \tau_0) \frac{\tau_0}{t} \tag{8}$$

$$T^3_{eff}(\mathbf{x}) = \left(\int_{\tau_0}^\infty dt T^6(\mathbf{x} + z\hat{\phi}, \tau_0) \left(\frac{\tau_0}{t}\right)^2 \right) \left(\int_{\tau_0}^\infty dt T^3(\mathbf{x} + z\hat{\phi}, \tau_0) \frac{\tau_0}{t} \right)^{-1} \tag{9}$$

This varient is similar to the *Path Dependent* prescription, except the temperature distribution now evolves with time through a Bjorken-like evolution ($\sim 1/t$)

Path Dependent - Pure Hydro

$$L(\mathbf{x}, \phi, \tau_0) = \frac{1}{T^3_{eff}(\mathbf{x})} \int_{\tau_0}^{\infty} dt T^3(\mathbf{x} + z\hat{\phi}, t) \tag{10}$$

$$T^3_{eff}(\mathbf{x}) = \left(\int_{\tau_0}^{\infty} dt T^6(\mathbf{x} + z\hat{\phi}, t) \right) \left(\int_{\tau_0}^{\infty} dt T^3(\mathbf{x} + z\hat{\phi}, t) \right)^{-1} \tag{11}$$

In the *Path Dependent - Pure Hydro* prescription, the time evolution of the medium is now governed by the full hydrodynamical simulation. This model of the effective length is the most sophisticated of the variants explored here.

The remaining two prescriptions for the effective length are similar to the first two prescriptions presented, only the hydrodynimcal model for the temperature is switched out for the number of participant density, n_{part}, which can be defined by the Glauber Model [12].

For each effective length, a probability is assigned to the likelihood of the length occurring. This allows us to model the complex structure of the QGP's geometry. The prescription we use for assigning these probabilities is outlined here.

We assume that the number of hard partons moving through the drop of QGP are proportional to the number of binary collision density (n_{coll}); ultimately the geometry of the drop of QGP will have varying shapes which suggests we should somehow be averaging over these shapes in our calculations. We use a procedure for obtaining the geometric average as outlined in [5].

2.3. Results of the Effective Length Prescription

The various effective length prescriptions described in eq. (4) to eq. (10) all differ from each other in subtle ways. Each prescription should ideally allow us to capture the true energy loss that occurs as a parton moves through the QGP. What we find in fig. 1, is that the probability distributions assigned to each of the effective lengths and temperatures vary substantially. This in turn implies that our final R_{AA} predictions differ from model to model. These highly variable results call into question the robustness of the effective length prescription used previously throughout the field.

3. Modifications to the DGLV Model

We now introduce a new framework for calculating the R_{AA}, moving away from the previous model that relied on effective lengths and temperatures to represent the QGP's complex dynamics. In the previous approach, averaging these quantities led to a wide spread of results with minor adjustments. In contrast, the new method models the scattering center using a power law defined by two parameters, ρ_0 and T_c.

$$\rho_{fit}(z) = \frac{\rho_0}{z} \theta(T_c - z) \tag{12}$$

where T_c is a thermalization cutoff. Each path that a parton follows through the QGP medium can be assigned a ρ_0 and T_c value. Many paths are then weighted and averaged over to calculate an R_{AA}.

3.1. Radiative Energy Loss

Following the work of Kolbe [7], we can write the scattering centre, $\bar{\rho}$, which forms as a probability distribution for the number of scattering cites, in terms of a density (ρ) for the number of scattering centres. Using the density ρ allows the energy loss formalism to become independent of effective lengths.

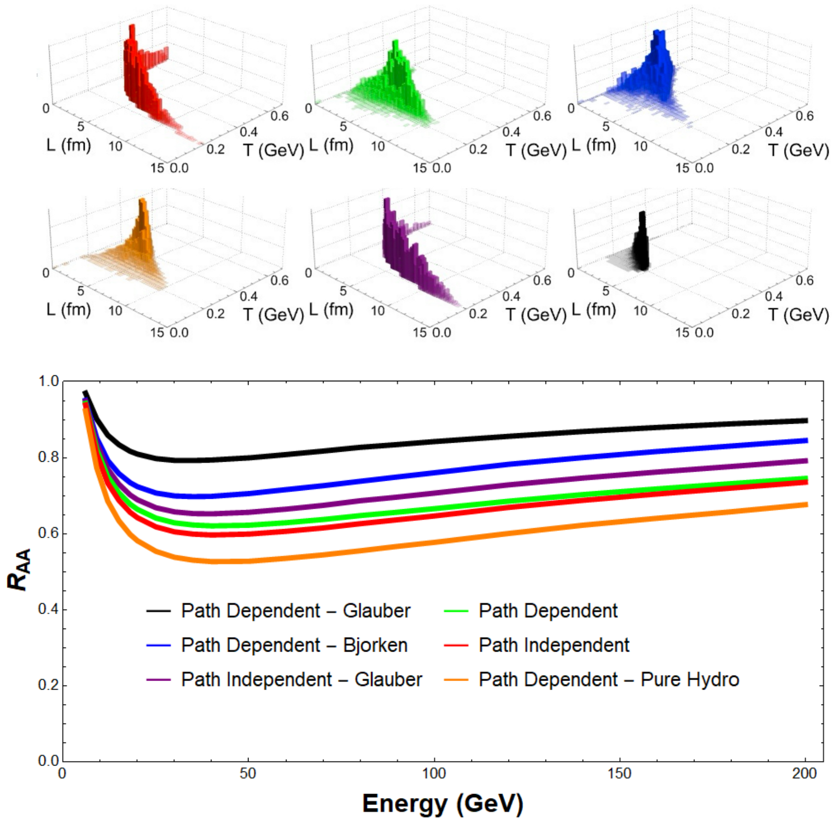

Figure 1: Top: Probability distributions (un-normalized) for effective lengths and temperatures across different models show significant differences. Bottom: R_{AA} vs. incoming bottom quark energy for various effective length prescriptions reveals a wide spread in predictions, indicating potential issues with the robustness of these models.

3.2. Modeling the Scattering Centre

Following the work of Faraday et. al. [9], we can obtain an expression for the scattering center (ρ) in terms of a temperature distribution (generated using hydrodynamical principles) as:

$$\rho(z) = \frac{4\zeta(3)(4+n_f)}{\pi^2} T^3(z) \tag{13}$$

Using ρ from eq. (13) in the modified formalism allows for the computation of the energy loss of a parton without the need for effective lengths or temperatures. This means no simplifications to the geometry of the QGP are needed to calculate an R_{AA}. However, this prescription becomes computationally expensive when calculating an average R_{AA} that takes into account many paths through the QGP medium. We thus use the *full* calculation that includes hydrodynamical temperature background as test to our power-law fit of eq. (12), the power-law fit allows for significant computational speed-up.

3.3. Results

Thus far, we have only calculated R_{AA} predictions based on radiative energy loss without accounting for the short path length correction. These preliminary results show strong agreement between the *full* calculation using the hydrodynamical background and the fitted background, as demonstrated in fig. 2.

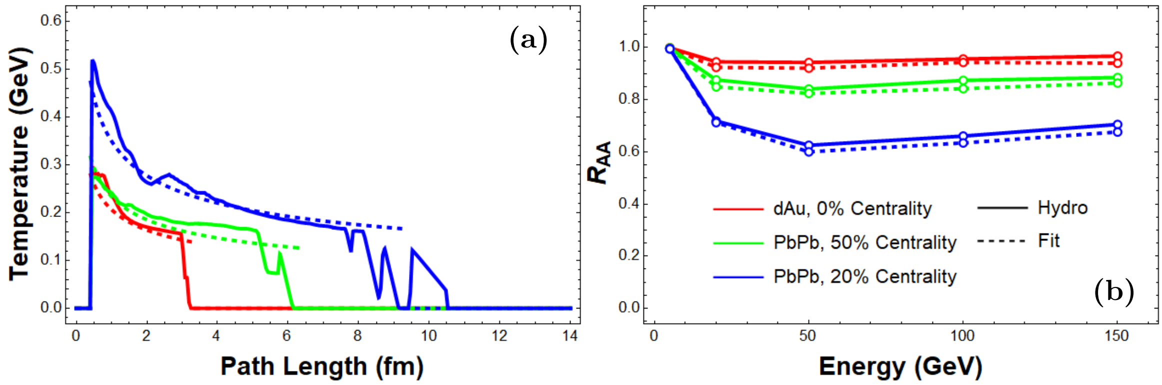

Figure 2: (a) Fitted temperature profiles (dashed lines) are calculated to match the hydrodynamic model's thermalization path length, with ρ_0 adjusted to equalize the areas under the curves. (b) Radiative R_{AA} comparison for bottom quarks shows that the fitted model aligns well with the full hydrodynamic calculation across different collision types.

4. Conclusion

The R_{AA} is an observable in heavy ion physics that is indicative of the amount of energy loss that occurs as partons move through the QGP. We are interested in modeling the R_{AA} observable, as directly measuring the Energy loss is not experimentally possible.

Previous attempts to model R_{AA} using effective length prescriptions have lacked robustness, with small changes leading to widely varying results. We have demonstrated alternative methods to calculate R_{AA}, which currently only consider radiative energy loss. The new approach, using a two-parameter power-law to fit scattering centers, accurately matches the predictions of a full hydrodynamical treatment. Future work aims to extend this model to include short path length corrections and elastic energy loss effects.

Acknowledgements

All three authors would like to thank the National Research Foundation and the SA-CERN collaboration for their support.

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